

On the Use of Very Small Pitot-Tubes for Measuring Wind Velocity.

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Introduction.

A recent paper by Stanton and Marshall* describes experiments made to determine the distribution of velocity close to the wall of a pipe through which air in a turbulent state is flowing. In order to measure the velocity of the air, Stanton used a very fine Pitot-tube placed in the pipe so that it faced the wind.

There is plenty of evidence that Pitot-tubes used in ordinary aeronautical work do provide an accurate measure of wind speed, the excess of pressure inside the tube over that in the surrounding air being $\frac{1}{2}\rho v^2$, where ρ is the density and v the velocity of the air. On the other hand, there are two factors which would lead one to expect that this law might not hold under the conditions which occurred in Stanton's experiment.

In the first place, the wind velocity is varying very rapidly across the mouth of the Pitot-tube. If, therefore, the formula $p = \frac{1}{2}\rho v^2$ be applied to determine v from the measured value of p , the exact position of the layer of fluid which would be flowing with velocity v , if it were undisturbed by the Pitot-tube, is a matter of doubt.

In Stanton's experiment this difficulty is accentuated, because, in order to obtain measurements very close to the wall, he uses a Pitot-tube which has only one side, the other being replaced by the wall itself. In such a tube, the " v " to be used in the formula $p = \frac{1}{2}\rho v^2$ might be expected to lie between zero and the velocity at the level of the outer wall of the Pitot-tube. This may be expressed by saying that the "effective distance" of the Pitot-tube from the wall is less than the total opening of the one side of the Pitot-tube. The "effective distance" is defined as the distance from the wall at which the air moves with velocity $\sqrt{(2p/\rho)}$, p being the pressure in the Pitot-tube.

In the second place, when a very small tube is used, or when wind speed is very low, one might expect that p would be proportional to v rather than to v^2 .

Stanton in his experiment gets over both these difficulties at once by first using his Pitot-tube in a pipe through which air is flowing in stream-line

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motion. Under these conditions the distribution of velocity in the pipe is known. The Pitot-tube can therefore be calibrated. The calibration consists in determining the "effective distance" of the Pitot-tube from the wall for various openings.

Stanton found that the "effective distance" of his Pitot-tube from the wall was connected with the total opening by a linear relation. It was found, however, that the straight line which represents the results does not pass through the origin, so that there appears to be a finite "effective distance" corresponding with the zero velocity which is known to occur at the wall. This result might have been anticipated.

It is possible to carry the matter further however. Suppose we assume that the pressure at the centre of Stanton's one-sided Pitot-tube is the same as that at the nose of a sphere, of the same diameter as the opening of the Pitot-tube, when placed in a stream moving with a velocity equal to that at the centre of the Pitot opening. The velocity at the centre of the Pitot opening, assuming stream-line flow, is $\bar{v}$, where

$$\bar{v} = v_0 \left\{ 1 - \frac{(a-d)^2}{a^2} \right\} = v_0 \frac{(2a-d)d}{a^2}, \quad (1)$$

where v_0 is the velocity at the centre of the pipe, a its radius, and $2d$ the breadth of the Pitot opening.

On the assumption made above, the pressure p in the Pitot-tube would be, by Stokes' law,

$$p = \frac{3}{2} \frac{\mu \bar{v}}{d} = \frac{3}{2} \mu v_0 \left\{ \frac{2}{a} - \frac{d}{a^2} \right\} \quad (2)$$

from (1), μ being the coefficient of viscosity.

On this assumption, therefore, it will be seen that the pressure tends to a constant value $3\mu v_0/a$ when d tends to zero. The "effective distance," which will be denoted by d' , corresponding with this pressure, is the distance from the wall at which the velocity is equal to $\sqrt{(2p/\rho)}$. Since the rate of shear close to the wall is $2v_0/a$, we have

$$\frac{2v_0}{a} d' = \sqrt{\left(\frac{2p}{\rho} \right)} = \sqrt{\left(\frac{6\mu v_0}{a\rho} \right)}$$

whence

$$d' = \sqrt{\left(\frac{3\mu a}{2\rho v_0} \right)}. \quad (3)$$

In Stanton's case the experiments were made with a pipe at three different velocities of v_0 , *i.e.*, 1910, 1140, and 740 cm. per second. Using the values $\mu/\rho = 0.148$, $a = 0.1345$ cm., equation (3) gives the following values for d' , namely, 0.0039, 0.0051, and 0.0063 cm.

The results of Stanton's calibration experiments are shown in fig. 1. In

this diagram the abscissæ represent the opening of the one-sided Pitot-tube while the ordinates represent the "effective distance." It will be seen that, when the opening tends to zero, the "effective distance" tends to 0.005 cm. approximately. This value lies within the range of the values calculated above.

Stanton obtains one mean calibration curve from his three sets of results

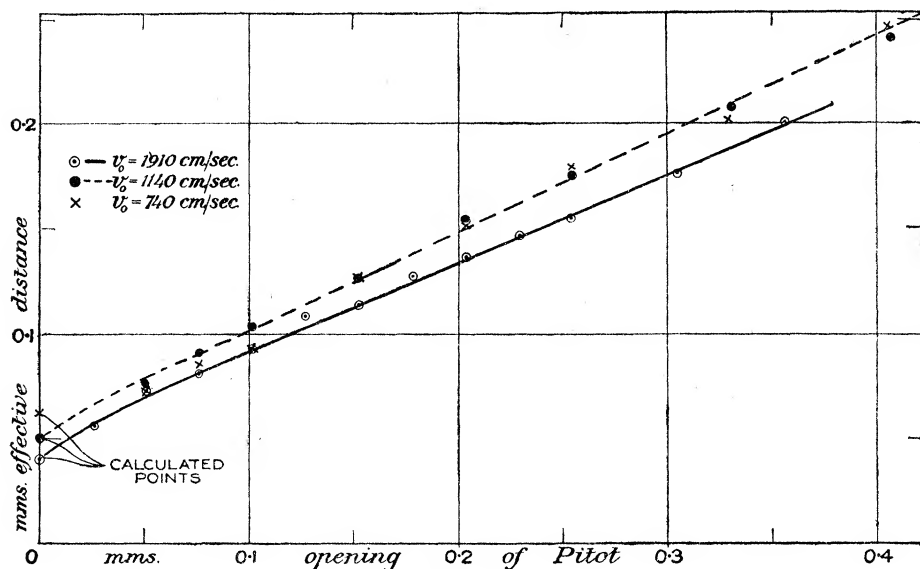


FIG. 1.

Diameter of pipe, 0.269 cm. Pitot-tube No. 3.

at different velocities. Formula (3), however, indicates that d' is not independent of v_0 . This might be expected from a consideration of the theory of dimensions. In fact, the points obtained by Stanton, when $v_0 = 1910$ and 1140 cm. per second respectively, do lie on different lines as shown in fig. 1, and the points where these lines cut the y axis correspond very fairly to the ones calculated from Formula (3).

It appears, therefore, that Stanton's results are explicable, if it is assumed that the v^2 law breaks down for small Pitot-tubes.

Investigation Undertaken in Present Paper.

In view of the results discussed above, it seemed of interest to examine the pressure differences between a Pitot and static-tube when placed in a fluid moving with very low velocity. It seemed probable that, for values of the velocity less than a certain fixed value, the pressure differences would cease to vary as the square of the speed. The present investigation was under-

taken with a view to determining this region, and, should sufficiently accurate measurements be possible, to determining the relation between pressure difference and velocity in this region.

A Pitot-tube of circular cross-section was mounted in the centre of a pipe through which water, in a state of stream-line motion, was flowing. A hole in the side of the pipe opposite the mouth of the Pitot-tube was connected to one side of a manometer and the Pitot to the other. The pressure differences registered by the manometer for various values of the mean velocity, down to as low a velocity as possible, were observed.

Since the velocity at the centre of a pipe, through which water is flowing in stream-line motion, is equal to twice the velocity of mean flow, the velocity at the centre of the pipe was found by measuring the volume of water ejected from the pipe in a given time.

Results and Conclusions.

As a result of the investigation, it is found that the law $p = \frac{1}{2}\rho v^2$ holds down to a velocity at the centre of the pipe of 6 cm. per second, *i.e.*, for values of $rv/\nu > 30$, where r is the radius of the Pitot-opening, ν the kinematical viscosity of the fluid, and v the velocity of the fluid at the centre of the Pitot-opening.

For values of $v < 6$ cm. per second, the pressure differences are greater than would be given by the above law, and we have a definite indication (see fig. 2) of a viscosity effect.

It was surmised that this viscosity effect might show itself in the form of an additional pressure, comparable with that at the nose of a sphere of radius equal to that of the Pitot opening, when placed in a stream moving with a velocity equal to that at the centre of the pipe. If this pressure be denoted by p_1 , we have, as before (see Introduction),

$$p_1 = \frac{3}{2} \frac{\mu v}{r}.$$

The results, in fact, give support to such a supposition. For, from the mean curve of fig. 2, we can determine the experimental values of the pressure differences for values of $v < 6$ cm. per second. Now, if we plot these experimental values against values of the pressure obtained from the formula

$$p = \frac{1}{2}\rho v^2 + p_1 = \frac{1}{2}\rho v^2 + \frac{3}{2} \frac{\mu v}{r},$$

the points are found to lie on a straight line through the origin (see Table II and fig. 3). The accuracy with which the points lie on this line is

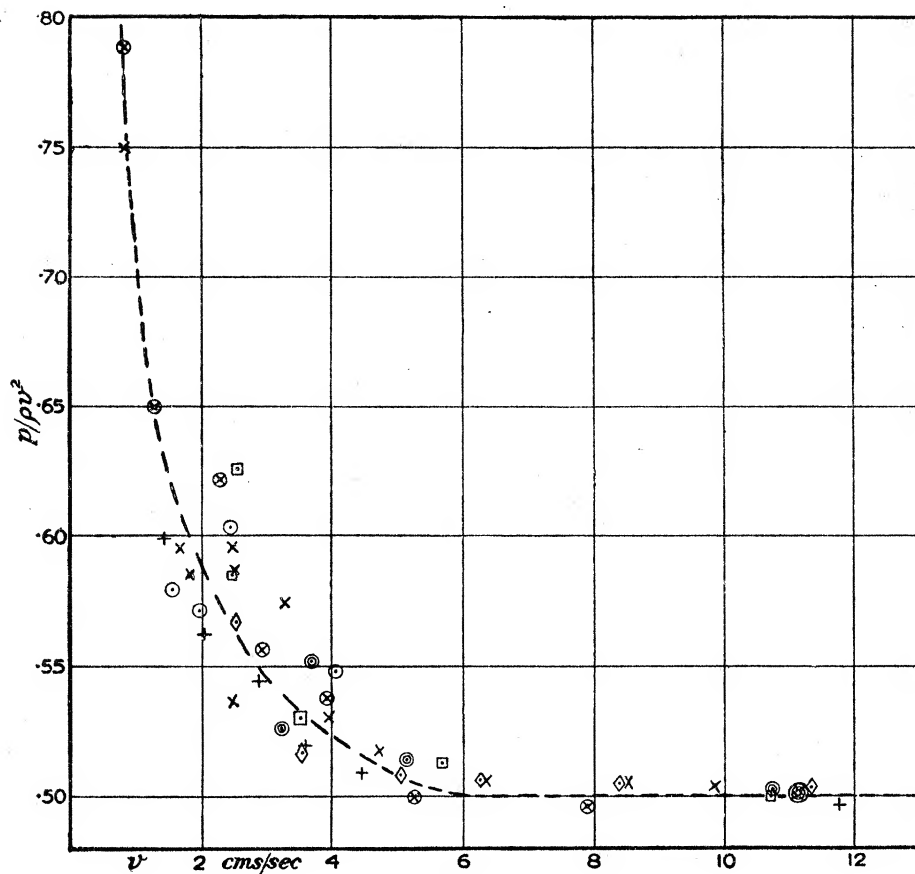


FIG. 2.

v = velocity at centre of pipe = twice mean velocity of outflow.
 p = pressure difference between Pitot and static tubes.
 Different symbols indicate different sets of readings (see Table I).

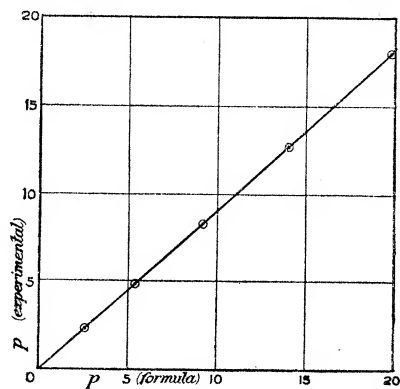


FIG. 3.

Ordinates mean of fig. 2. Abscissæ from formula.

probably fortuitous, but a viscosity effect of the type suggested is clearly demonstrated.

It is interesting to note that much of Stanton's work is carried out in a

Table I.

v = velocity at centre of pipe = twice mean velocity of outflow.

p = pressure difference between Pitot and static-tubes.

(For use of symbol attached to each set of readings in Column T, see fig. 2.)

T.	v cm. per sec.	p dynes per sq. cm.	$p/\rho v^2$.	T.	v cm. per sec.	p dynes per sq. cm.	$p/\rho v^2$.
° C.				° C.			
13	0·82	0·53	0·788	14·6	2·42	3·42	0·585
	1·35	1·19	0·650		2·55	4·08	0·626
	2·35	3·43	0·622		3·52	6·59	0·530
	3·12	5·41	0·557		5·67	16·50	0·513
	3·96	8·44	0·538		10·65	58·80	0·500
	5·24	13·70	0·500	□			
	7·90	30·75	0·494				
	11·14	62·10	0·500	15	0·84	0·53	0·750
⊗					1·70	1·72	0·595
14	2·51	3·56	0·568		1·84	1·98	0·585
	3·58	7·25	0·566		2·44	3·56	0·596
	5·09	13·19	0·509		2·46	3·30	0·546
	6·23	19·65	0·506		2·51	3·70	0·587
	8·38	35·50	0·505		3·32	6·33	0·575
	11·34	64·90	0·504		3·96	8·30	0·530
◇					4·74	11·60	0·517
14	1·58	1·15	0·580		6·34	20·30	0·506
	1·95	2·175	0·572		8·42	35·90	0·506
	2·45	3·625	0·604	×	9·87	49·30	0·504
	4·07	9·10	0·549		11·11	62·00	0·502
	11·12	62·00	0·500	15	1·41	1·19	0·598
⊙					2·05	2·375	0·565
14·2	3·21	5·41	0·526		2·87	4·48	0·545
	3·695	7·52	0·552		3·60	6·725	0·520
	5·15	13·60	0·514		4·44	10·01	0·509
	10·73	58·05	0·503	+	11·76	68·75	0·496
⊙							

TABLE II.

Experimental values of p from mean curve of fig. 2.

Theoretical values of p given by $p = \frac{1}{2}\rho v^2 + \frac{3}{2}\mu v/r$.

v cm./sec.	p exper.	p theor.
2	2·35	2·6
3	4·9	5·4
4	8·4	9·2
5	12·7	14·0
6	18·0	19·8

region for which $rv/\nu < 30$, where in this case $2r$ is taken to be the breadth of the rectangular Pitot-tube.

For example, taking No. 3 Pitot and the case when $r = 0.0064$ cm. and $v = 400$ cm. per second, we have

$$rv/\nu = 20 \text{ approx.}$$

Description of the Experiment.

A brass pipe, some 70 cm. long and 1.1 cm. diameter, was fitted with a small Pitot-tube whose cross-section was 0.1 cm. in diameter, and whose axis coincided as nearly as possible with the axis of the pipe. This tube was connected to one side of a Chattock tilting gauge. Slightly behind the mouth of the Pitot-tube, a hole in the side of the pipe was connected to the other side of the gauge (see fig. 4).

As water was the fluid flowing through the pipe and contained in

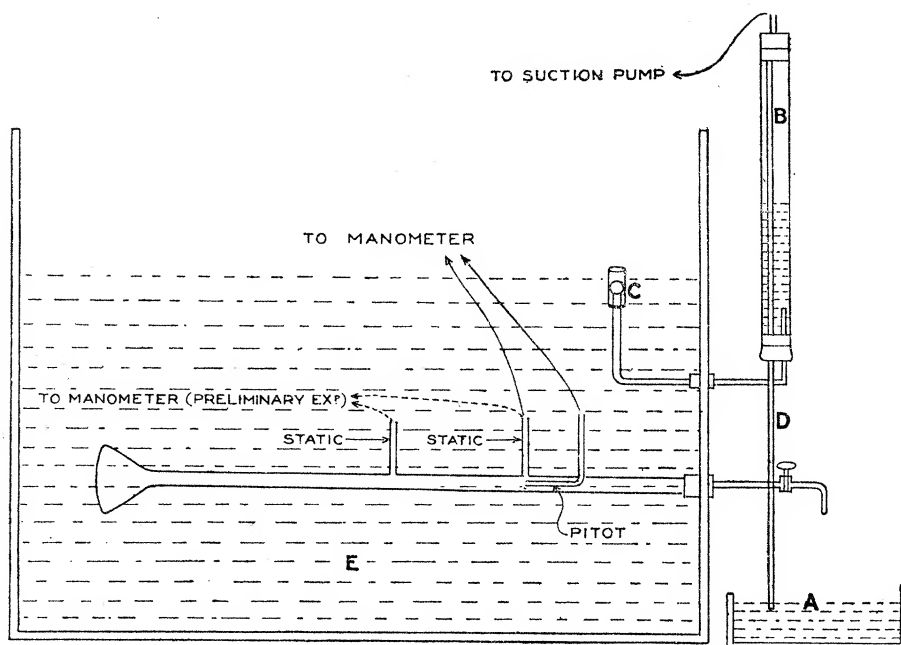


FIG. 4.—Diagram of apparatus. Not to scale.

the connecting tubes and upper halves of the manometer bulbs, brine could not be used as the heavy liquid in the manometer. Instead, nitro-benzene, of density 1.207, was used, and, as the difference in density between the two liquids in the manometer bulbs was now only 0.207, the sensitivity of the manometer was increased about five times, and was such that one

division of the circular scale represented a pressure difference between the two sides of 0.00134 mm. of water.

The pipe was provided with a smooth converging mouthpiece and placed in a trough of water, E, the level of which was kept constant by means of a circulating pump. The end of the pipe projected through the side of the trough, and was provided with a stop-cock, by means of which the flow could be regulated. The water from the pipe was received in a trough, A, from which it was withdrawn by means of a suction pump up a long tube, D, into a thin glass cylinder, B. As soon as the water in A fell below the mouth of the tube D, the pressure of the water that had been drawn up into B opened the steel ball-valve at C, and the water in B flowed back into the trough E.

With this method of circulating the water, it was found that, though the temperature of the water in the trough varied from day to day, it remained reasonably constant over the period of 2-3 hours needed to take a set of readings. The water was allowed to circulate for some time before readings were taken, and a record of the temperature was kept.

Preliminary Experiment.

As a preliminary experiment, the pressure differences between the static-tube and a second static-tube nearer the mouth of the pipe were measured for different rates of mean flow, in order that the critical velocity for the pipe might be determined.

In the tabulated results (see Table III) the pressures have been corrected for temperature in accordance with the laws of dynamical similarity. The results are illustrated in fig. 5. It will be seen that, for velocities at the centre of the pipe of less than 12 cm. per second, the velocity is proportional to the pressure differences, and hence stream-line motion exists in this region. The principal experiment was therefore carried out in this region.

Principal Experiment.

The Pitot-tube and the static-tube opposite its mouth were now connected to the two sides of the manometer, and a series of readings taken for velocities varying from 12 to 0.8 cm. per second. The water ejected from the pipe in a certain time, determined by a stop-watch, was weighed. In the manometer used, one revolution of the wheel corresponded to a pressure difference of 0.0648 cm. of water. The circular scale on the wheel was divided into hundredths, and readings could be depended upon to one division. The difference of density between the two liquids in the

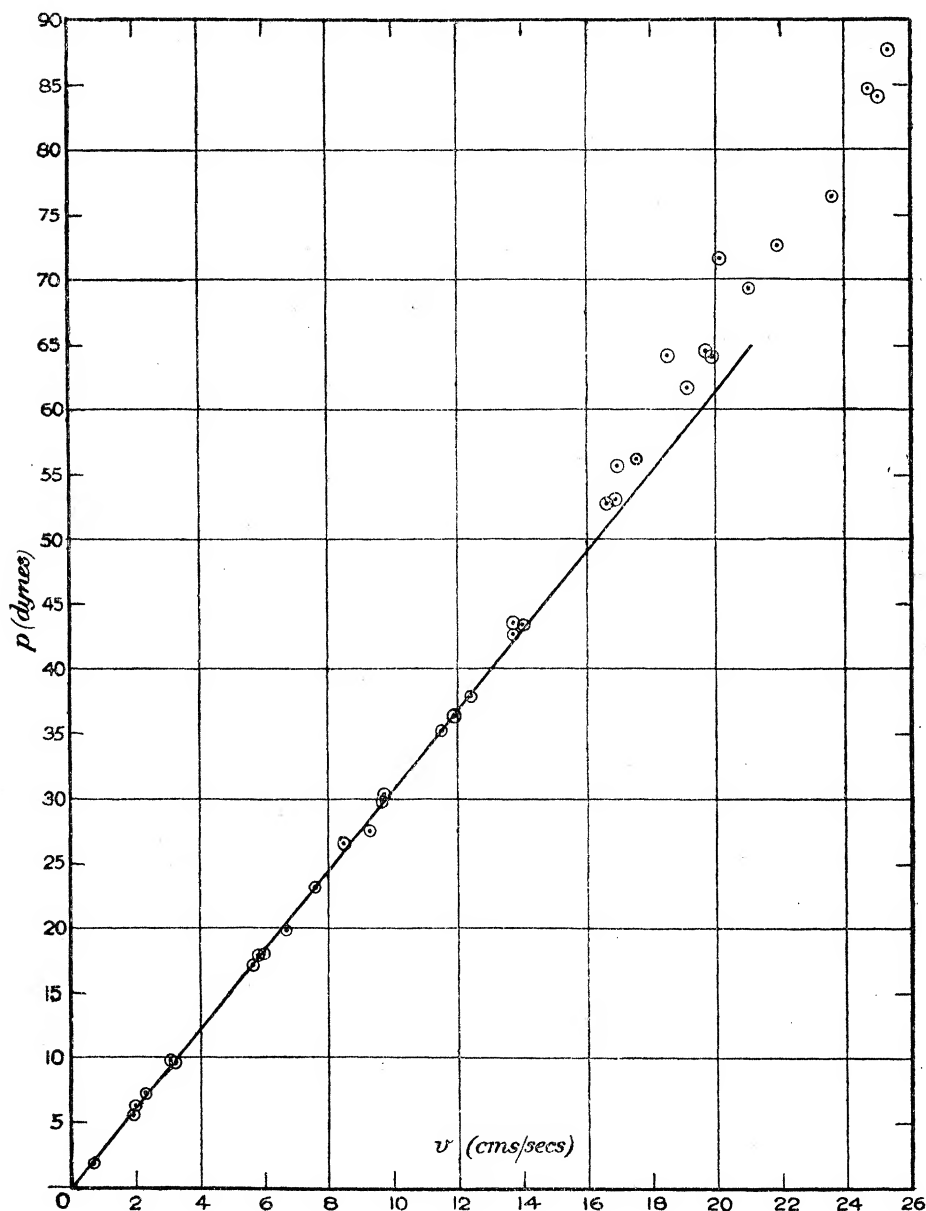


FIG. 5.

v = velocity at centre of pipe = twice mean velocity of outflow.

p = pressure difference for a length 18.5 cm. of pipe at 12° C.

manometer bulbs was 0.2072. Thus, given a reading h in revs. of the manometer wheel, the pressure p in dynes was given by the formula

$$p = 0.2072 \times g \times 0.0648h = 13.17h.$$

Table III.

 v = velocity at centre of pipe = twice mean velocity of outflow. p = pressure difference for a length 18.5 cm. of pipe at 12° C. (the pressures have been corrected for temperature).

v cm. per sec.	p dynes per sq. cm.	v cm. per sec.	p dynes per sq. cm.
1.87	5.7	16.78	55.7
3.22	9.6	18.52	64.1
6.66	19.8	20.10	71.6
11.94	36.4	20.80	69.2
15.16	47.4		
16.60	52.8	2.30	7.2
19.12	61.6	5.60	17.1
19.88	64.1	9.69	29.9
24.70	84.8	12.40	38.0
		17.57	56.1
5.80	17.9	21.05	69.4
8.47	26.5	25.00	84.1
13.70	42.7		
19.70	64.6	0.69	1.8
21.95	72.6	2.00	6.3
25.40	87.8	3.12	9.75
		5.85	18.1
1.93	5.75	7.60	23.1
6.00	18.0	9.24	27.6
9.80	30.4	11.50	35.2
13.69	43.5	16.84	53.1
14.00	43.3	23.62	76.5

Discussion of the Results.

The results are tabulated in Table I and illustrated in fig. 2 above, where $p/\rho v^2$ is plotted against velocity, and we obtain a mean curve of the usual "scale effect" type. From the fact that, down to so low a velocity as 6 cm. per second, the experimental points lie so consistently on the line $p/\rho v^2 = \frac{1}{2}$, we see that the pressure difference between the Pitot and static-tubes used in the experiment does give a very accurate measure of the square of the velocity for values of $v > 6$ cm. per second. The results obtained for values of $v < 6$ cm. per second give a very definite indication of a viscosity effect on the relation between p and v in this region.

It will be seen from Table I that, with decrease of v , the pressure differences registered by the manometer become exceedingly small. The degree of approximation obtained can be estimated from the distribution of the points in the figure.

For different sets of readings, the temperature of the water varied from 13° to 15° C., and this variation would be expected to influence the results in the region just mentioned. But the pressure differences were too small for any effect to be detected.

It seems probable that more accurate data for this purpose could be obtained by repeating the experiment, using, instead of water in the pipe, a liquid of, say, five times the density of water. Then, according to the laws of dynamical similarity, the viscosity effect should show itself over a range of velocities for which accurate results could be obtained.

In conclusion, I should like to express my hearty thanks to Mr. G. I. Taylor, F.R.S., at whose suggestion the experiment was undertaken, and to whom I am indebted for much valuable help and advice during the course of the work.

On the Susceptibility of Feebly Magnetic Bodies as Affected by Compression.

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Introduction.

This communication undertakes an investigation of the complex problem which is presented by the effects of mechanical stress upon the susceptibility, retentivity and other properties of magnetic substances. The present experiments are confined to compressive stress, and its effects upon the susceptibility of certain rock specimens. It was felt that an investigation into this part of the subject might possibly throw some light upon the susceptibility of the earth's crust as affected by the enormous forces with which it has to contend, and their variations. It is unfortunate that, owing to the nature of rock specimens, the compressive stress has been limited to about 1200 kgm. per square centimetre, but, nevertheless, some interesting results have been obtained, and these are recorded in the present paper.

All the specimens are in the form of short bars about 4 cm. in length, whose cross-section is either square, being 1 cm. across each side, or 1 cm. in diameter; and, throughout the work, the compressive stress has been applied in the direction coinciding with the length of the bar. The susceptibility has been measured (*a*) in the direction of the stress and (*b*) at right angles thereto.